

THE FASTEST ROTATING PULSAR: A STRANGE STAR?

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ABSTRACT

According to the observational limits on the radius and mass, PSR 1937+21 is probably a strange star, or some neutron star equations of state should be ruled out, if we alternatively suggest that a dipole magnetic field (rather than a quadrupole one) is relevant to its radio emission. It is conjectured that PSR 1937+21 and PSR 1257+12 (and possibly some or all of the isolated millisecond pulsars) may have weaker dynamo-originated magnetic field due to a less effective field magnification process.

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Pulsars are conventionally modeled as neutron stars, but a strange star model for pulsars is also proposed (see a review, e.g., by Xu et al.¹). Therefore it is of great importance to find observationally unambiguous features to distinguish strange stars from neutron stars. Up to now there are three hopeful ways to identify a strange star, which are based upon: 1, that strange stars (or neutron stars containing significant quantities of strange matter) may rotate more rapidly²; 2, that the mass-radius relations of strange stars are in striking contrast to that of neutron stars³; 3, the eccentric behavior of the quark surface condition¹ which results in distinguishable signatures in magnetospheric and polar radiation. Recently, Kapoor & Shukre⁴ suggest to constrain the equations of state of neutron stars by the Rankin's experiential line⁵ of the core emission of radio pulsars, and find that the pulsar masses $M \leq 2.5M_\odot$ and radii $R \leq 10.5$ km, indicating that pulsars are strange stars. However Kapoor & Shukre's work⁴ has at least two unseemly points: 1, the polar cap is defined for aligned rotators (rather than for orthogonal rotators) in their calculation; 2, the Rankin line is not doubtless true due to many observational and statistical uncertainties. By redefining the polar cap for orthogonal rotators, we find we can not get a conclusion that pulsars are strange stars, or that some neutron stars equations of state are ruled out, if the Rankin line is used to constrain pulsar masses and radii. Nevertheless, when applying this method to the fastest rotating pulsar, PSR 1937+21, we conclude that the pulsar may be a strange star, or at least some equations of state should be ruled out. In order to find solid evidence to shown that PSR 1937+21 is a strange star, it is strongly suggested to measure precisely the maximum rate of position-angle swing by further polarization observations.

Now, we improve Kapoor & Shukre's computation by removing the first unseemliness. In a spherical coordinate with magnetic axes μ being chosen as the z-axes, one has⁶ for a vector position \mathbf{r} in a dipole field line denoted by parameter λ ,

$$r = \lambda \frac{cP}{2\pi} \sin^2 \theta, \quad (1)$$

where θ is the polar angle, P the rotation period, c the speed of light, and λ characterizes the sorts of field lines. Based on Eq.(1), the angle θ_μ between μ and the direction of magnetic field at \mathbf{r} reads,

$$\cos \theta_\mu = \frac{2\lambda - 3\mathcal{R}}{\sqrt{\lambda(4\lambda - 3\mathcal{R})}}, \quad (2)$$

where $\mathcal{R} \equiv \frac{2\pi r}{cP} \ll 1$ for points near pulsar surface. Expand function θ_μ in the vicinity of $\mathcal{R} = 0$, one comes to

$$\theta_\mu = \frac{3}{2\sqrt{\lambda}} \mathcal{R}^{1/2} + \frac{3}{8\lambda\sqrt{\lambda}} \mathcal{R}^{3/2} + O(\mathcal{R}^{5/2}). \quad (3)$$

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Assuming $\lambda = \lambda_0$ for the last-open-field lines (note: λ_0 is a function of inclination angle α), one can obtain the beam radius ρ for the radiation at an emission height $h = r - R$ (R is the radius of pulsar), neglecting the terms being equal or higher than $\mathcal{R}^{3/2}$ in Eq.(3),

$$\rho = \frac{3}{2} \sqrt{\frac{2\pi}{\lambda_0 c}} \frac{r^{1/2}}{P^{1/2}} = \frac{1^\circ.24}{\sqrt{\lambda_0 P}} \sqrt{\frac{r}{10\text{km}}}. \quad (4)$$

For core emissions ($r \gtrsim R$), which are supposed to be originated from regions near pulsar surfaces⁵, general relativistic effects are not negligible due to the spacetime curvature⁷. Two such effects, squeezing of the dipole magnetic field and bending of the radio wave, can be represented approximately by two factors^{7, 4}, f_{sqz} and f_{bnd} , respectively, so that the beam radius ρ can be re-written as

$$\rho = \frac{3}{2} \sqrt{\frac{2\pi}{\lambda_0 c}} \frac{r^{1/2}}{P^{1/2}} = \frac{1^\circ.24}{\sqrt{\lambda_0 P}} \sqrt{\frac{r}{10\text{km}}} f_{\text{sqz}} f_{\text{bnd}}, \quad (5)$$

where $f_{\text{sqz}} = (1 + \frac{3GM}{2c^2 r})^{-1/2}$, $f_{\text{bnd}} = \frac{1}{3}(2 + (1 - \frac{2GM}{c^2 r})^{-1/2})$, G is the gravitation constant, M the pulsar mass.

Now we are to calculate the parameter λ_0 of last-open-field lines, assuming that only the plasma within the light cylinder with radius $r_{\text{lc}} = \frac{cP}{2\pi}$ can corotates with the star, i.e., field lines that penetrate beyond the light cylinder should be open. λ_0 is generally a function of inclination angle α . Nevertheless, one can get simple formulae of λ_0 in two special cases. Obviously, $\lambda_0(\alpha = 0) = 1$ for an aligned rotator according to Eq.(1). Another formula of λ_0 can be easily reached for an orthogonal rotator ($\alpha = 90^\circ$). The conal angle θ_0 of the null surface, where the magnetic fields are perpendicular to magnetic axes $\vec{\mu}$, can be obtained from Eq.(2) by setting $\theta_\mu = 90^\circ$,

$$\theta_0 = \sin^{-1} \sqrt{\frac{2}{3}}. \quad (6)$$

Let $r \cos \theta_0 = r_{\text{lc}}$, one finds

$$\lambda_0(\alpha = 90^\circ) = \frac{3\sqrt{3}}{2}, \quad (7)$$

which is 2.6 times that of $\lambda_0(\alpha = 0^\circ)$.

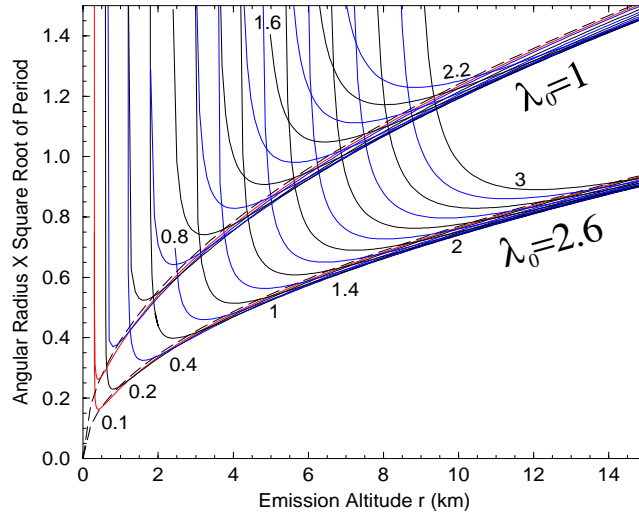


FIG. 1.— The calculated results based on Eq.(5) show the variations of $\rho \times \sqrt{P}$ (in unit of degree \cdot second^{1/2}) as a function of emission altitude r for different pulsar masses ranging from $0.1M_\odot$ to $\gtrsim 2M_\odot$. We present results of $\lambda_0 = 1$ (aligned rotators) and $\lambda_0 = 3\sqrt{3}/2$ (orthogonal rotators) for two limits; an actual value for a pulsar with artificial inclination angle α should be between those two corresponding values. The dashed lines are calculated by Eq.(11), which are not relevant to pulsar mass.

Following the representation by Kapoor and Shukre⁴, we calculate the value, $\rho \times \sqrt{P}$, as a function of emission altitude r for pulsars with core emission according to Eq.(5). The calculated results are shown in Fig.1. Contrary

to the result of Kapoor & Shukre⁴, it is clear in Fig.1 that the Rankin line (i.e., $\rho \times \sqrt{P} = 1.225^\circ \text{ s}^{1/2}$) can not effectively put any limit on the equations of state of neutron stars or conduce toward any indication that pulsars are strange stars if we assume that Rankin's samples are of nearly orthogonal rotators. Nevertheless, we may still get some valuable conclusions from above calculation for some special pulsars, for instance, the fastest rotating millisecond pulsar, PSR 1937+21.

The rotation period of PSR 1937+21 is $P = 1.558 \text{ ms}$, the smallest one of pulsars observed, and the rate of period change is $\dot{P} = 10^{-19} \text{ s} \cdot \text{s}^{-1}$. The radius of light cylinder of the star is $r_{lc} = \frac{cP}{2\pi} = 74 \text{ km}$. The corotation radius, defined by the balance of gravitational force and the centrifugal force, is $r_c = (\frac{GM}{4\pi^2})^{1/3} P^{2/3} = 20(M/M_\odot)^{1/3} \text{ km}$, where M is the mass of PSR 1937+21. Therefore the pulsar radius R satisfies

$$R < \min(r_{lc}, r_c) = 20(M/M_\odot)^{1/3} \text{ km}. \quad (8)$$

Another circumscription of the stellar radius R and the mass M may arise from the inclusion of general relativistic effect⁴. The pulse width $\Delta\phi$ of PSR 1937+21 is about 10° at 600 MHz and 1.4 GHz (EPN database), which is difficult to be explained geometrically if we suggest PSR 1937+21 has a radius of 10 km for canonical pulsars⁸. Gil propose thus a model⁹ in which a quadrupole magnetic field geometry is assumed. However an alternative conjecture to overcome the difficulty is that PSR 1937+21 may have *unusual* small radius. Taking the simplest proposing that $\alpha = 90^\circ$ and impact angle $\beta = 0$, we know $\Delta\phi = 2\rho$, and therefore $\rho\sqrt{P} = 0.2^\circ \text{ s}^{1/2}$. According to the results in Fig.1, it is found that

$$M < 0.2 M_\odot \quad \text{and} \quad R < 1 \text{ km}. \quad (9)$$

These *stringent* limits have to result in the conclusion that PSR 1937+21 is a *strange star* rather than a neutron star because of the strikingly different mass-radius relations.

However, the conclusion above is *not* so solid. In fact there are two models proposed to account for the inter-pulse emission¹⁰: single-pole model and double-pole model. Three observational facts⁹ favour a double-pole model of inter-pulse origin of PSR 1937+21. 1, The gradient of the position angle at inter-pulse center has the same value as at the main-pulse center. 2, The separation of the main-pulse and the inter-pulse is nearly 180° . 3, The intensities of the main-pulse and the inter-pulse are roughly equal. The observed maximum rate of position angle swing at the main-pulse center is $(d\psi/d\phi)_{\max} \approx 3$. In the standard rotating vector model¹¹, the impact angle β should be

$$\beta = \sin^{-1} \left[\frac{\sin \alpha}{(d\psi/d\phi)_{\max}} \right] \sim 19.5^\circ, \quad (10)$$

and the beam radius ρ thus is derived as

$$\rho = \cos^{-1} \left[\cos \beta - 2 \sin \alpha \sin(\alpha + \beta) \sin^2 \frac{\Delta\phi}{4} \right] \sim 20.1^\circ, \quad (11)$$

if we take $\alpha = 90^\circ$, $(d\psi/d\phi)_{\max} = 3$ and $\Delta\phi = 10^\circ$. In this case, we find $\rho\sqrt{P} = 0.79^\circ \text{ s}^{1/2}$, and the limits thus are

$$M < 2.4 M_\odot \quad \text{and} \quad R < 11.5 \text{ km} \quad (12)$$

based on Fig.1.

It is well known that the masses and radii of rotating magnetized neutron stars are equation-of-state dependent. The radius of a neutron star with a limiting-mass is the smallest; a smaller mass, a slower rotation and/or a lower inner magnetic field would cause the its radius to be larger^{12, 13, 14}. Therefore the observational deduction of Eq.(12) should have some implications for proof-testing the equations of state available. Five equations of state have been focused¹² in the calculation for rapidly rotating neutron stars, the results of which are listed in Table 1 by interpolating between the tabulated points in Table 9-23 of Cook et al.¹² for angular frequency $\Omega = 4 \times 10^3 \text{ s}^{-1}$ (i.e., $P = 1.558 \text{ ms}$). We see that the equations labeled as ‘‘L’’ and ‘‘M’’ should be ruled out by the limits of Eq.(12). Certainly, we should also keep in mind that more equations of state may be killed by inclusion of the inner magnetic field effect¹⁴, which is an interesting topic in the future study.

Furthermore, two observational uncertainties may result in a derivation of stronger limits on the mass and radius of PSR 1937+21. The first is that observation would give a less steep position angle gradient (i.e., a smaller value of $(d\psi/d\phi)_{\max}$ than reality) due to smearing of finite sampling time, to the frequency dispersion in pulse arrive time¹⁵, and to the interstellar scattering⁹ which may bring on a stretching out of the longitude

scale. $(d\psi/d\phi)_{\max} > 3$ in fact. A larger $(d\psi/d\phi)_{\max}$ (thus a smaller β according to Eq.(10)) makes for a rough equality of the intensities of the main-pulse and inter-pulse. The second is that PSR 1937+21 might not be exactly an orthogonal rotator (i.e., α is not precisely 90°). It is possible that $\alpha < 90^\circ$. Both these two uncertainties lead to a smaller value of $\rho\sqrt{P}$. We thus take $(d\psi/d\phi)_{\max}$ and α as two free parameters to calculate $\rho\sqrt{P}$. The calculated results are shown in Fig.2. It is clear that stronger limits than of Eq.(12) may rule out more neutron star equations of state, and probably conduce toward a strange star model for PSR 1937+21, based on Fig.1 and Fig.2.

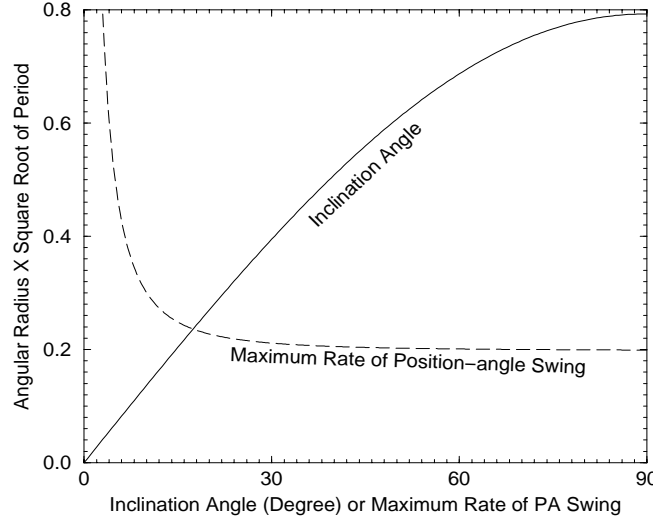


FIG. 2.— $\rho\sqrt{P}$ (beam angular radius times square root of period) of PSR 1937+21 for possible different inclination angles α or maximum rates of position-angle swing $(d\psi/d\phi)_{\max}$.

In Fig.2 we find $\rho\sqrt{P}$ is sensitively dependent on $(d\psi/d\phi)_{\max}$ if $(d\psi/d\phi)_{\max} \lesssim 10$. $\rho\sqrt{P} = 0.6$ if $(d\psi/d\phi)_{\max} = 4$ and $\alpha = 90^\circ$, in which case the limits of Eq.(12) should be modified as $M < 1.4 M_\odot$ and $R < 6.6$ km. These limits inevitably lead to the conclusion that PSR 1937+21 is a strange star. Because of the observational uncertainties of $(d\psi/d\phi)_{\max}$ and α and of the importance of distinguishing a strange star in nature, we seriously table a proposal of detecting more accurate polarization signal from PSR 1937+21 to get an actual value, especially, of $(d\psi/d\phi)_{\max}$. This could be possibly by means of reducing the sampling time and increasing the total observation time.

TABLE 1
THE TOTAL GRAVITATIONAL MASS-ENERGY M AND THE
CIRCUMFERENTIAL RADII R AT EQUATOR OF ROTATING ($P = 1.558$ ms)
NEUTRON STARS WITH MAXIMUM MASS FOR FIVE KIND OF EQUATIONS
OF STATE

* See Table 2 of Cook et al.¹² for details.

EOS*	M/M_\odot	$R(\text{km})$
A (Reid soft core)	1.6604	8.82
L (Mean field)	2.7263	14.98
M (Tensor interaction)	1.8298	19.08
AU (AV14 + UVII)	2.1433	9.78
FPS (UV14 + TNI)	1.8069	9.88

If PSR 1937+21 is a strange star with small radius ($R < 10$ km), the old formulae¹⁶ to calculate the surface magnetic field B and dipole magnetic moment μ_m should be modified as

$$\begin{aligned} B &= \left(\frac{c^3 \rho P \dot{P}}{5\pi R}\right)^{1/2} \sim 9.3 \times 10^{19} R_5^{-1/2} (P \dot{P})^{1/2} \text{ gauss}, \\ \mu_m &= \left(\frac{c^3 \rho P \dot{P} R^5}{20\pi}\right)^{1/2} \sim 4.6 \times 10^{29} R_5^{5/2} (P \dot{P})^{1/2} \text{ gauss cm}^3, \end{aligned} \quad (13)$$

since the density $\rho \sim 5 \times 10^{15} \text{ g cm}^{-3}$ has a very modest variation with radial distance of strange star¹⁷, $M \sim \frac{4}{3}\pi R^3 \rho$, and the moment of inertia $I \sim \frac{8}{15}\pi \rho R^5$, where R_5 is the stellar radius in 10^5 cm (1 km). We see from Eq.(13) that the calculated magnetic moment is strongly relevant to the radius although the observation-determined surface magnetic field is weakly relevant. For instance, if the radius of PSR 1937+21 is only 1 km, $B = 1.2 \times 10^9$ gauss, but $\mu_m = 5.8 \times 10^{18}$ gauss cm³ (typical μ_m for millisecond pulsars is assumed to be 10^{26} gauss cm³!). This property favors the assumption that dipole magnetic structure dominates in the radio emission region.

Millisecond pulsars are currently believed to be of recycled-origin of normal pulsars which are spin down enough. However, motivated by the study of planet formation around PSR 1257+12, Miller & Hamilton¹⁸ suggest that some and perhaps all isolated millisecond pulsars may have been born with high spin rates and low magnetic fields instead of having been recycled by accretion. This is understandable if we assume part or all of isolated millisecond pulsars are strange stars with smaller radii (thus smaller masses): 1, Turbulent convection in nuclear matter¹⁹ or in strange quark matter²⁰ should be less prosperous in a proto-pulsar with lower mass than with higher mass in the Kelvin-Helmholtz cooling phase, the dynamo-created magnetic field is thus weaker. 2, Small radius favors faster rotation since the centrifugal force become smaller while the gravitational force to be larger (see Eq.(8)). A bimodal distribution of pulsar periods and magnetic fields may arises from various kind of progenitors as well as complex mechanisms of supernova exploration.

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